



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

EARTHQUAKE RESISTANT DESIGN OF FOUNDATIONS

Lecture 22: Seismic Response of Pile Foundations-2

B.K. MAHESHWARI

Professor, Dept. of Earthquake Engineering, IIT Roorkee

E-mail: bk.maheshwari@eq.iitr.ac.in



ERD of Foundations - Modules

1. Introduction
2. Shallow Foundations
- 3. Pile Foundations**
4. Well Foundations
5. SSI for Deep Foundations



Module - 3

Pile Foundations



Module -3: Chapters

1. Piles for Static Vertical Load
2. Lateral Loads on Piles
3. Dynamic Analysis of Piles
4. ~~Design for Lateral and Dynamic Loads~~
Seismic Response of Piles
5. Codal Provisions

Chapter 4: Seismic Response of Pile Foundation

- **Introduction**
- **Basic Concepts**
- **Tajimi's Theory (Tajimi 1969)**
- **Novak's 3-D Model (Nogami and Novak 1976 & 1977)**
- **Dynamic Stiffness and Damping of Piles (Novak 1974)**
- **Seismic Response of End Bearing Pile (Gazetas 1984)**

Novak's 3-D Model (Nogami and Novak 1976 & 1977)

In this theory, analysis is performed separately for vertical and horizontal mode of vibration. When vertical mode of vibration is considered, horizontal displacement is neglected and vice-versa. Thus coupling between two modes of vibration are not considered. The theory assumes soil as linear, elastic, homogeneous and isotropic layers with frequency independent hysteretic type material damping.

For horizontal mode of vibration, this theory is the same as given by Tajimi except that in later case viscous damping is considered for the soil medium instead of hysteretic type considered in former case. Thus in the absence of material damping both theories are exactly the same for horizontal vibration. Like Tajimi's theory, in this case also, analysis is performed employing three-dimensional wave propagation technique. For vertical motion, only one wave equation is needed and solution is obtained in closed form providing complex soil stiffness for axial vibration.

Novak's 3-D Model (Nogami and Novak 1976 & 1977) (Continued...)

(i) Vertical Vibration:

(a) Complex Soil Stiffness:

Resistance force of soil, in vertical motion can be expressed as

$$p(z) = 2\pi\mu \sum_{n=1}^{\infty} \alpha_n W_n \sin(h_n z) \quad (3a)$$

where W_n denotes the amplitude of the vertical displacement in n th mode and dimensionless resistance factor α_n is given by

$$\alpha_n = (q_n r_0) (1 + iD_s) \frac{K_1(q_n r_0)}{K_0(q_n r_0)} \quad (3b)$$

$$q_n^2 = \frac{[\eta^2 + i\{D_v(\eta^2 - 2) + 2D_s\}]h_n^2 - (\omega/V_s)^2}{1 + iD_s} \quad (3c)$$

$$h_n = \frac{\pi}{2H} (2n - 1) \quad n = 1, 2, 3, \dots \quad (3d)$$

$$\eta = \frac{V_L}{V_s} = \sqrt{\frac{2(1-\nu)}{(1-2\nu)}} \quad (3e)$$

Novak's 3-D Model (Nogami and Novak 1976 & 1977) (Continued...)

(i) Vertical Vibration:

(b) Complex Stiffness of the pile-soil system :

$$k = \frac{EA}{r_0} \bar{k} \quad (4a)$$

where the dimensionless stiffness parameter is given by

$$\bar{k} = (r_0/H) \left(2 \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\bar{h}_n^2 - \bar{\lambda}^2 + \gamma \alpha_n} \right) \quad (4b)$$

$$\bar{h}_n = H h_n; \quad \bar{\lambda}^2 = \frac{m H^2}{EA} \omega^2 \quad \text{and} \quad \gamma = 2\pi \frac{\mu H^2}{EA} \quad (4c)$$

In above equations D_v and D_s are the hysteretic damping ratios associated with the volumetric and shear strains, respectively and are considered frequency independent. ν is the Poisson's ratio of soil. m and A are the mass per unit length and cross-sectional area of the pile, respectively.

Novak's 3-D Model (Nogami and Novak 1976 & 1977) (Continued...)

(ii) Horizontal Vibration:

(a) Complex Soil Stiffness:

Soil reaction in this mode can be expressed as

$$p(z) = \pi\mu \sum_{n=1}^{\infty} \alpha_{hn} U_n \sin(h_n z) \quad (5a)$$

where dimensionless horizontal resistance factor for n^{th} mode

$$\alpha_{hn} = r_0 \left[(1 + iD_s) h_n^2 - (\omega/V_s)^2 \right] T_n \quad (5b)$$

Where,

$$T_n = \frac{4K_1(q_n r_0)K_1(s_n r_0) + s_n r_0 K_1(q_n r_0)K_0(s_n r_0) + q_n r_0 K_1(s_n r_0)K_0(q_n r_0)}{q_n K_1(s_n r_0)K_0(q_n r_0) + s_n K_1(q_n r_0)K_0(s_n r_0) + q_n s_n r_0 K_0(q_n r_0)K_0(s_n r_0)} \quad (5c)$$

$$q_n^2 = \frac{(1 + iD_s) h_n^2 - (\omega/V_s)^2}{\eta^2 + i\{(\eta^2 - 2)D_v + 2D_s\}} \quad \text{and} \quad s_n^2 = \frac{(1 + iD_s) h_n^2 - (\omega/V_s)^2}{1 + iD_s} \quad (5d)$$

Novak's 3-D Model (Nogami and Novak 1976 & 1977) (Continued...)

(ii) Horizontal Vibration:

(b) Complex Pile-Head Stiffness:

Though it is possible to get pile head stiffness in closed form but it can be derived from the expression for lateral displacement (after applying boundary condition to determine unknown constants as needed to calculate horizontal, rocking or cross stiffness). Expression for lateral displacement is

$$u(z) = A \sin(\lambda z) + B \cos(\lambda z) + C \sinh(\lambda z) + D \cosh(\lambda z) - \sum_{n=1}^{\infty} \frac{\pi \mu \alpha_{hn} (A F_{1n} + B F_{2n} + C F_{3n} + D F_{4n})}{E I h_n^4 - m \omega^2 + \pi \mu \alpha_{hn}} \sin(h_n z) \quad (6a)$$

Where,

$$\lambda = (m \omega^2 / E I)^{1/4} \quad (6b)$$

Where A , B , C and D are integration constants determined by the boundary conditions at the ends of the pile. Also dimensionless functions F (i.e. F_{1n} , F_{2n} , F_{3n} and F_{4n}) are needed to expand trigonometric functions of argument (λz) into a Fourier sine series of argument $(h_n z)$ and thus known.

Comparison of Simplified Approaches

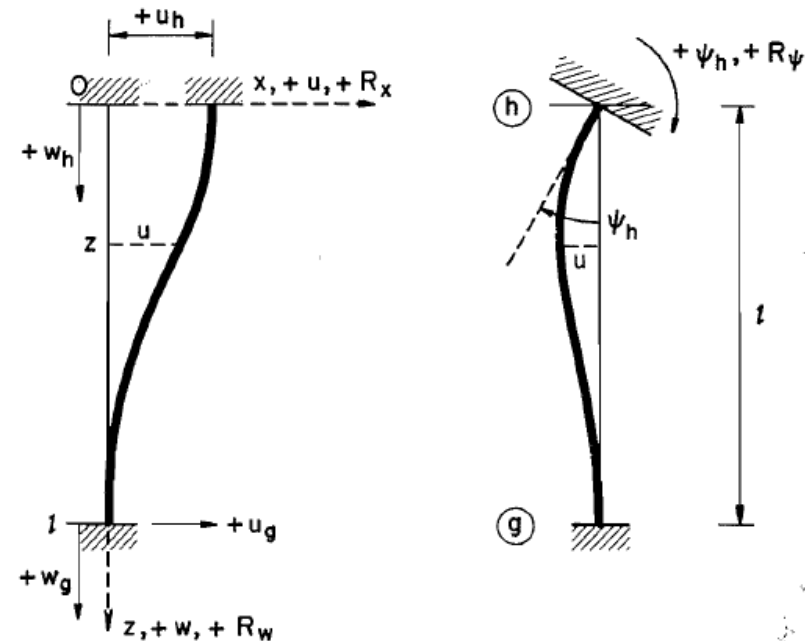
S. No.	Theory - Proposed by	Applicability* Single Pile / Pile Group	Mode of Vibration Vertical / Horizontal/both	Pile-Tip Condition** End-Bearing / Floating	Assumption / Concept
1.	Poulos (1968)	Pile Group	Considered only static loads in both directions	Floating	Soil is elastic, homogeneous, isotropic. Solutions in closed form.
2.	Penzien (1964)	Pile Group	Horizontal only	End-Bearing	Nonlinear discrete dynamic model for piles through clay. Numerical analysis. Difficult to incorporate radiation damping and inertia of soil.
3.	Tajimi (1969)	Single Pile	Horizontal only	End-Bearing	Soil is modeled as linear viscoelastic stratum. Viscous damping for soil.
4.	Novak (1974)	Pile Group	Horizontal only	End-Bearing	Soil is modeled as a generalized Winkler's model possessing inertia and capability to dissipate energy.
5.	Nogami & Novak (1976, 1977)	Single Pile	Both Vertical and Horizontal, separately	End-Bearing	Rigorous method (linear), based on three-dimensional wave equations. Hysteretic damping of soil.
6.	Plane-Strain Soil Model (1978)	Pile Group	Both Vertical and Horizontal, separately	End-Bearing, Floating piles if bedrock is at finite depth	No strain on the plane perpendicular to the axis. An extension of Novak (1974) and model at No. 5.
7.	Blaney et al. (1976)	Single Pile	Both Vertical & Horizontal with full coupling	End-Bearing	Finite elements to reproduce the soil around the pile and a consistent boundary matrix to simulate the far field. Nonlinear soil behavior can be considered.
8.	Wolf (1978)	Pile Group	Both Vertical & Horizontal with full coupling	End Bearing, extended for floating piles.	Extension of theory of Blaney (No. 7) for pile group and floating piles.
9.	Kaynia and Kausel (1982)	Pile Group	Both Vertical & Horizontal with full coupling	End Bearing as well as floating piles	A very rigorous and versatile approach. 3-D linear analysis, Fully accounts for pile-soil-pile interaction

Dynamic Stiffness and Damping of Piles, Novak (1974)

Dynamic response of footings and structures supported by piles can be predicted if dynamic stiffness and damping generated by soil-pile interaction can be defined. An approximate analytical approach based on linear elasticity is presented, which makes it possible to establish the dimensionless parameters of the problem and to obtain closed-form formulas for pile stiffness and damping. All components of the motion in a vertical plane are considered; that is, horizontal as well as vertical translations and rotation of the pile head. The stiffness and damping of piles are defined in such a way that the design analysis of footings and structures resting on piles can be conducted in the same way as is applied in the case of shallow foundations

Dynamic Stiffness and Damping of Piles, Novak (1974)

Consider first the cases in which the pile is excited by horizontal translation of its head or lower end and by rotation of its head in the vertical plane (Fig). Only lateral displacement of pile elements is considered. The effect of the static axial force is not accounted initially but is considered separately later.



Displacements and reactions of piles.

Dynamic Stiffness and Damping of Piles, Novak (1974)

Horizontal Translation and Rotation in the Vertical Plane

When a pile element dz undergoes a complex horizontal displacement $u(z, t)$ at height z , it will meet a horizontal soil reaction

$$G(S_{u1} + iS_{u2})u(z, t)dz \quad (1)$$

in which G = shear modulus of soil and $i^2 = -1$. Parameters S_{u1} and S_{u2} are functions of dimensionless frequency $a_o = r_o \omega / V_s$ and depend on Poisson's ratio ν . The real and imaginary parts of the complex function

$$S_u(a_o, \nu) = (S_{u1}(a_o, \nu) + iS_{u2}(a_o, \nu)) \quad (2)$$

$$\text{or} = 2\pi a_o * \frac{\frac{1}{\sqrt{q}} H_2^{(2)}(a_o) H_1^{(2)}(x_o) + H_1^{(2)}(x_o) H_1^{(2)}(a_o)}{H_0^{(2)}(a_o) H_2^{(2)}(x_o) + H_0^{(2)}(x_o) H_2^{(2)}(a_o)}$$

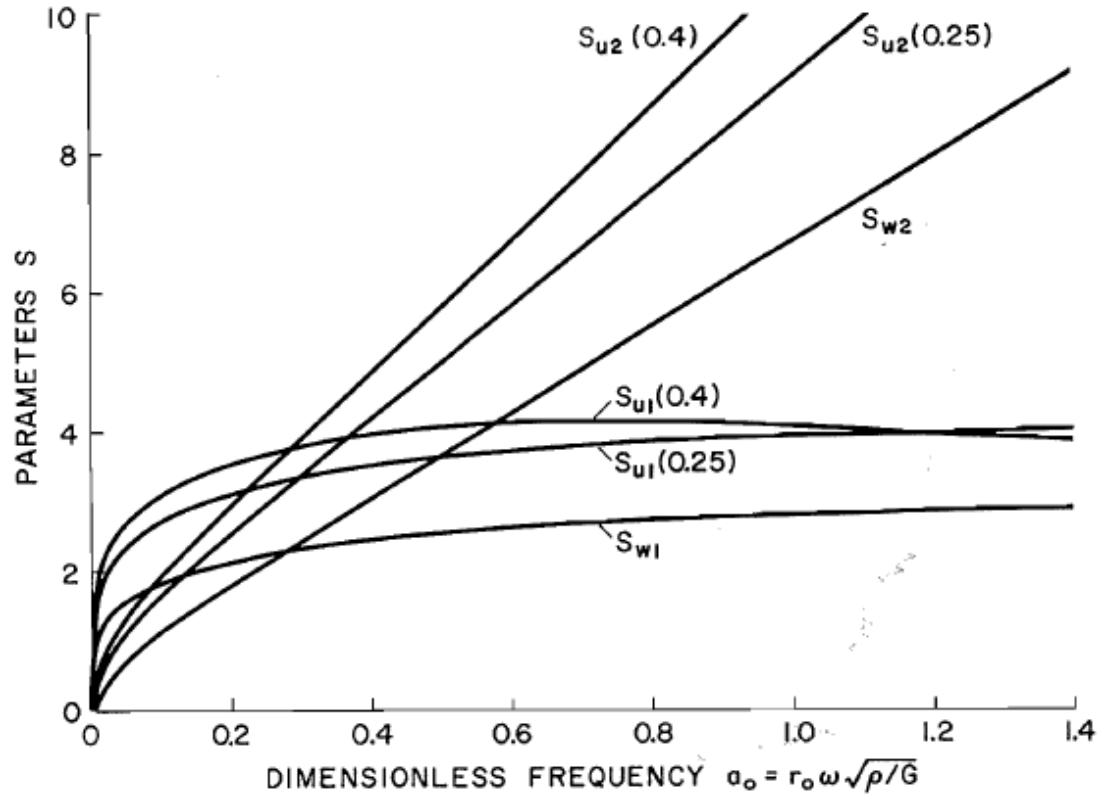
where r = pile radius, ω = frequency

Dynamic Stiffness and Damping of Piles, Novak (1974)

Horizontal Translation and Rotation in the Vertical Plane

In Eq. (2), $q = (1 - 2\nu)/(2(1 - \nu))$, $x_o = a_o\sqrt{2}$ and $H_n^{(2)}$ = Hankel functions of the second kind of order n . Equation (2) was derived by Baranov 1967. Parameters S_{u1} and S_{u2} were calculated for several values of Poisson's ratio in Beredugo and Novak 1972, in which polynomial approximations to them are also given. The parameters S are shown in Fig.

Dynamic Stiffness and Damping of Piles, Novak (1974)



Dynamic Stiffness and Damping of Piles, Novak (1974)

Vertical Vibrations

The dynamic reactions of the pile pertinent to vertical motion of the pile head can be obtained using the same assumptions for soil as in the previous case. Assume further that the lower end of the pile is fixed. Then, the vertical soil reaction acting at height z on pile element dz is (Baranov 1967; Novak and Beredugo 1972):

Then, the vertical soil reaction acting at height z on pile element dz is (Baranov 1967; Novak and Beredugo 1972):

$$p(z, t)dz = G(S_{w1} + iS_{w2})w(z, t)dz \quad (3)$$

Where, G = shear modulus of soil surrounding the pile; and

Dynamic Stiffness and Damping of Piles, Novak (1974)

$$S_{w1} = 2\pi a_o \frac{J_1(a_o)J_o(a_o) + Y_1(a_o)Y_o(a_o)}{J_o^2(a_o) + Y_o^2(a_o)} \quad (4)$$

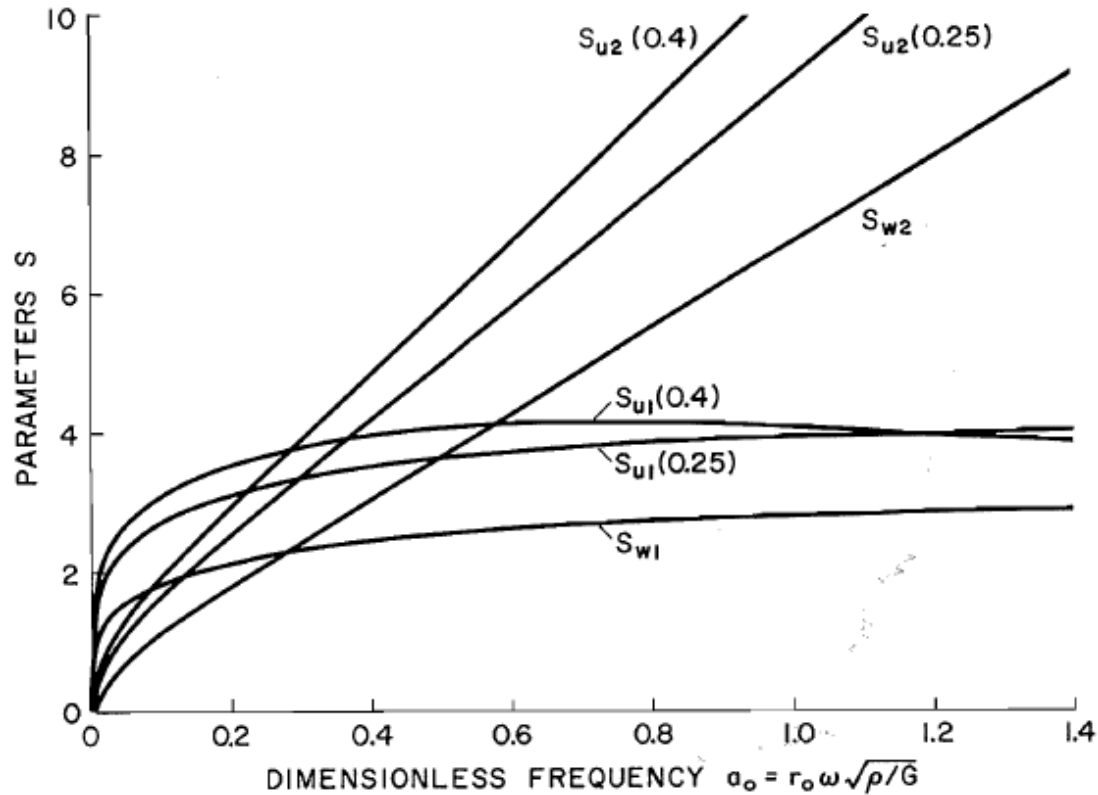
$$S_{w2} = \frac{4}{J_o^2(a_o) + Y_o^2(a_o)} \quad (5)$$

Where,

$J_o(a_o), J_1(a_o)$ = Bessel functions of the first kind of order zero and one

$Y_o(a_o), Y_1(a_o)$ = Bessel functions of the second kind of order zero and one

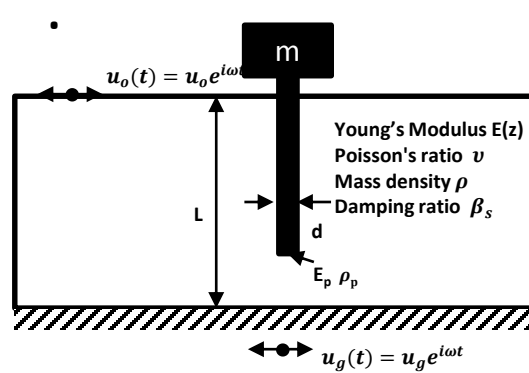
Dynamic Stiffness and Damping of Piles, Novak (1974)



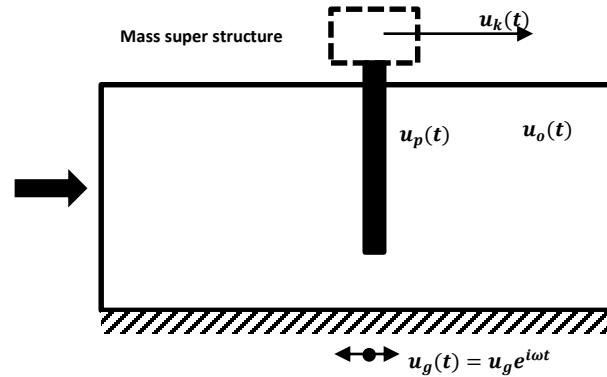
Gaetas's (1984) Theory for End Bearing Single Pile

A numerical study is presented of the dynamic response of end-bearing piles embedded in a number of idealized soil deposits and subjected to vertically propagating harmonic S-waves. Results, for both 'kinematic' and 'inertial' interaction, are offered in the form of dimensionless graphs and formulae covering a wide range of excitation frequencies and of crucial material and geometric parameters. Practical aspects of the evaluation of the influence of piles on the effective seismic excitation of a structure are discussed and a case history illustrates the usefulness of the presented results.

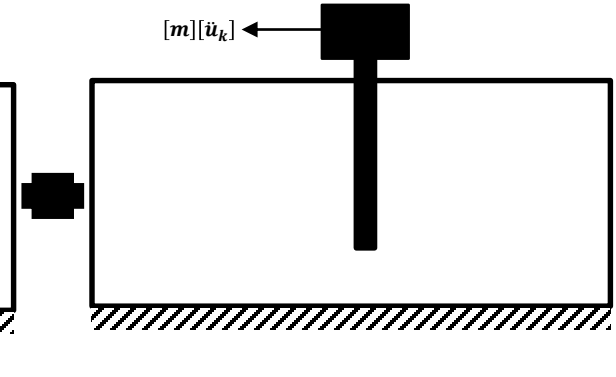
Gaetas's (1984) Theory for End Bearing Single Pile



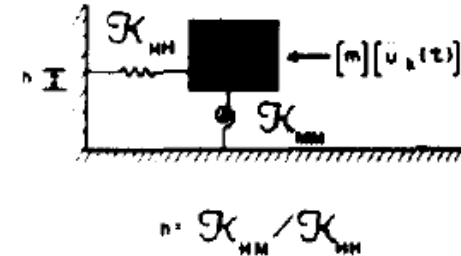
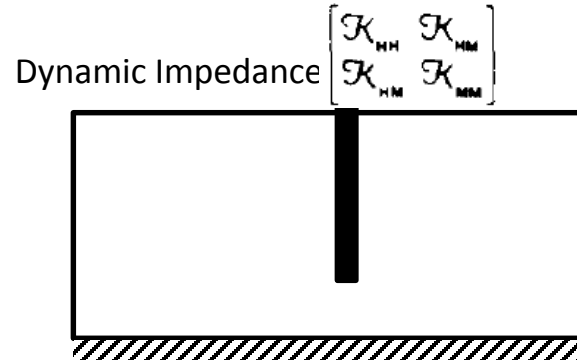
Geometry of soil-pile-structure



Kinematic Interaction



Inertial Interaction



$$n = K_{HM} / K_{MH}$$

Gaetas's (1984) Theory for End Bearing Single Pile

Parametric results: kinematic interaction

In the absence of a pile, a vertically incident S-wave would induce only horizontal displacements in the free-field soil. For a base rock motion $u_g(t) = u_g \exp(i2\pi ft)$ and a homogeneous soil stratum, the one-dimensional 'amplification' theory would give for the steady-state free-field displacement at ground surface level

$$u_o(t) = u_o \exp(i2\pi ft) \quad (1a)$$

$$\frac{u_o}{u_g} = \frac{2}{e^{iql} + e^{-iql}} \quad (1b)$$

(L is the stratum thickness and $q^2 = 4\pi^2 f^2 / [V_s^2(1 + 2i\beta_s)]$, where V_s and β_s are the S-wave velocity and the internal (hysteretic) damping in the soil: $i^2 = -1$.) If $\beta_s = 0$, equation (1b) yields $\frac{u_o}{u_g} = 1/\cos(2\pi fL/V_s)$ which tends to infinity (resonance) at the natural shear frequencies of the stratum, $f_n = (2n - 1) V_s/4L$, $n = 1, 2, 3 \dots$

Gaetas's (1984) Theory for End Bearing Single Pile

A cylindrical pile diffracts the incident and reflected one-dimensional vertical S-waves, thereby modifying the 'free' wave field. As a result, the horizontal displacement at the top the pile, $u_p(t) = u_p \exp(i2\pi ft)$ differs from $u_o(t)$ of equation (1). In addition, the pile top experiences a rotation, $\phi_p(t) = \phi_p \exp(i2\pi ft)$. It is convenient to portray the effects of kinematic interaction by introducing the displacement and rotation kinematic interaction factors.

$$I_u = \frac{u_p}{u_o} \text{ and } I_\phi = \frac{\phi_p r_o}{u_o} \quad (2)$$

and also, the displacement and rotation kinematic amplification factors

$$A_u = \frac{u_p}{u_g} \text{ and } A_\phi = \frac{\phi_p r_o}{u_g} \quad (3)$$

where $r_o = d/2$ is the radius of the pile, while U_p and U_o are the amplitudes of horizontal displacement of the pile top and the free-field ground surface, respectively.

References

1. **Gazetas G. (1984), “Seismic Response of End-Bearing Piles”, Soil Dynamics and Earthquake Engineering. Vol. 3, No.2, pp. 82-93**
2. **Maheshwari B.K. and Watanabe H. (2009), “Seismic Analysis of Pile Foundations using Simplified Approaches”, Int. Journal of Geotechnical Engineering, Vol. 3, pp. 387-404.**
3. **Novak M. (1974), “Dynamic Stiffness and Damping of Piles”, Canadian Geotechnical Journal, Vol. 11, No. 4, pp. 574-598.**

Thank You

